Leśniewski and the Logic of Names

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1. Introduction

Referential concepts in conceptual realism are based on a logic of proper and common names as parts of quantifier phrases. This conceptualist logic of names is similar to Leśniewski's logic of names in that the category of names in Leśniewski's system also contains common as well as proper names.

Leśniewski's logic is different, however, in that names do not occur as parts of quantifier phrases but are of the same category as object variables. Leśniewski described his logic of names as "ontology," apparently because it was to be the initial level of a theory of types, which Leśniewski called semantic categories.

Leśniewski's logic of names has been used for years as a framework in which to interpret and reconstruct various doctrines of medieval logic. Recently, I developed an alternative interpretation and reconstruction of medieval logic in terms of the framework of conceptual realism. It is relevant therefore to see how, or in what respect, Leśniewski's logic of names is similar to our conceptualist logic of names.

In fact, as we will see, Leśniewski's logic of names can be completely interpreted in — and in that sense is reducible to — our conceptualist logic of names.

We will first briefly describe Leśniewski's logic of names and then the simple logic of names that is part of our more comprehensive formal ontology for conceptual realism.

We will then explain how Leśniewski's system can be interpreted within our logic and how certain oddities of Leśniewski's system can be explained in terms of our logic where those oddities do not occur. We will then explain how the logic of classes as many is developed as an extension of the simple logic of names.

The logic of names of Leśniewski's ontology and our ontology of conceptual realism provide an example, incidentally, of how different parts of a formal ontology can be developed independently of, or even prior to, the construction of a comprehensive system all at once.

2. Leśniewski's Logic of Names

In Leśniewski's logic of names, as in our conceptualist logic, there is a distinction between:

(1) shared, or common names, such as 'man', 'horse', 'house', etc., and even the ultimate superordinate common name 'thing', or 'object';

(2) unshared names, i.e., names that name just one thing, such as proper names; and

(3) vacuous names, i.e., names that name nothing.

There is a categorial difference between names in Leśniewski's logic and names in our conceptualist logic, however. In Leśniewski's logic names belong to the same category as the object variables, which means that they are legitimate substituends for those variables in first-order logic.

In our conceptualist logic, names belong to a category of expressions to which quantifiers are applied and that result in quantifier phrases such as 'every raven', 'some man', 'every citizen', etc.

The one primitive of Leśniewski's logic, aside from logical constants, is the relation symbol 'ε' for **singular inclusion**, which is read as the copula '*is (a)*', as in 'John *is a* teacher', where both 'John' and 'teacher' are names.

Using 'a', 'b', 'c', etc., as object constants or variables for names, the basic formula of the logic is '$a\ \varepsilon\ b$', where 'a', b' are names, shared, unshared, or vacuous.

A statement of the form '$a\ \varepsilon\ b$' is true in Leśniewski's logic if, and only if 'a' names exactly one thing and that thing is also named by 'b', though 'b' might name other things as well.

Thus, our example of 'John is a teacher', can be written as

$$\text{John } \varepsilon \text{ teacher.}$$

Identity is not a primitive logical concept of Leśniewski's system, as it is in our conceptualist logic, but is defined instead as follows:

$$a = b \quad =_{\mathbf{df}} \quad a\ \varepsilon\ b \wedge b\ \varepsilon\ a.$$

That is, '$a = b$' is true in Leśniewski's logic if, and only if, 'a' and 'b' are unshared names that name the same thing.

This seems like a plausible way to understand '$a = b$', except that then, where 'a' is a shared or vacuous name, '$a = a$' is false.

In fact, because there are necessarily vacuous names, such as the complex common name 'thing that is both square and not square', the following is provable in Leśniewski's logic:

$$(\exists a)(a \neq a),$$

which does not seem at all like a plausible thesis. Of course, this means that

$$(\forall a)(a = a)$$

is not a valid thesis in Leśniewski's system.

Leśniewski does include a weak notion of identity, which is defined as follows

$$a \circ b \ =_{\mathbf{df}} \ (\forall c)(c \ \varepsilon \ a \leftrightarrow c \ \varepsilon \ b).$$

This notion, of course, means that a and b are co-extensive, not identical.

But then, Leśniewski did insist on his logic being extensional, and not intensional, in which case $a \circ b$ does amount to a kind of identity when a is either a shared or unshared name.

But then, in that case all vacuous names, such as 'Pegasus' and 'Bucephalus' are identical in this weak sense. It also means that Leśniewski's ontology is not an appropriate framework for tense and modal logic, or for intensional contexts in general.

Another valid thesis of Leśniewski's logic is,

$$\varphi(c/a) \to (\exists a)\varphi(a),$$

which seems counter-intuitive when 'c' is a vacuous name. The following, for example, would then be valid

$$\neg(\exists b)(b = Pegasus) \to (\exists a)\neg(\exists b)(b = a),$$

and therefore, because the antecedent is true, so is the consequent, which says that something is identical with nothing.

Perhaps these oddities can be explained by interpreting the Leśniewski's first-order quantifiers substitutionally rather than referentially. That, however, is not how Leśniewski understood his logic of names, which, as we have said, he also called ontology.

A logic that interprets the quantifiers of its basic level substitutionally, rather than referentially, would be an odd sort of formal ontology.

Does that mean that a name would be required for every object in the universe, including, e.g., every grain of sand and every microphysical particle?

We should also note that Leśniewski's epsilon symbol, 'ε', for singular inclusion should not be confused with the epsilon symbol, '$\in$', for membership in a set. In particular, whereas the following:

$$a \ \varepsilon \ b \to a \ \varepsilon \ a,$$
$$a \ \varepsilon \ b \wedge b \ \varepsilon \ c \to a \ \varepsilon \ c,$$

are both theorems of Leśniewski's system, both are invalid for membership in a set.

Finally, the only nonlogical axiom of ontology — i.e. the only axiom in addition to the logical axioms and inference rules of first-order predicate logic *without* identity — assumed by Leśniewski was the following:

$$(\forall a)(\forall b)[a \,\varepsilon\, b \leftrightarrow (\exists c) c \,\varepsilon\, a \wedge (\forall c)(c \,\varepsilon\, a \to c \,\varepsilon\, b) \wedge (\forall c)(\forall d)(c \,\varepsilon\, a \wedge d \,\varepsilon\, a \to c \,\varepsilon\, d)].$$

This axiom alone does not suffice for the *elementary* logic of names, however, i.e., for Leśniewski's logic of names as formulated independently of the higher-order part of Leśniewski's framework. But it has been shown that adding the following two axioms to the one above does suffice:

(Compl) $(\forall a)(\exists b)(\forall c)[c \mathrel{\varepsilon} b \leftrightarrow c \mathrel{\varepsilon} c \wedge c \not\varepsilon\, a]$,

(Conj) $(\forall a)(\forall b)(\exists c)(\forall d)[d \mathrel{\varepsilon} c \leftrightarrow d \mathrel{\varepsilon} a \wedge d \mathrel{\varepsilon} b]$.

Expressed in terms of our conceptualist logic, where names are taken to express name (or nominal) concepts, what these axioms stipulate is that there is a complementary name concept corresponding to any given name concept, and, similarly, that a conjunctive name concept corresponds to any two name concepts with singular inclusion taken conjunctively.

3. A Conceptualist Logic of Names

The simple logic of names of conceptualism can be described as a version of an identity logic that is free of existential presuppositions regarding singular terms — i.e., free object variables and expressions that can be properly substituted for such.

The logic also contains both absolute and relative quantifier phrases, i.e., relative quantifier phrases such as $(\forall xA)$ and $(\exists xA)$, as well as absolute quantifier phrases such as $(\forall x)$and $(\exists y)$, which are read as $(\forall x Object)$ and $(\exists y Object)$, respectively.

We will continue to use x, y, z, etc., with or without numerical subscripts, as object variables. We will now also use A, B, C, with or without numerical subscripts, as name (or "nominal") variables.

As explained in the previous lecture, complex names are formed by adjoining relative clauses to names, and we use '/', as in 'A/φ' to represent the adjunction of a formula φ to the name A (which may itself be complex).

Thus, e.g., the quantifier phrase representing reference to a house that is brown would be symbolized as

$$(\exists x House/Brown(x)).$$

We continue to take the universal quantifier, $\forall$, the (material) conditional sign, $\rightarrow$, the negation sign, $\neg$, and the identity sign, $=$, as primitive logical constants, and assume the others to be defined in the usual way.

The absolute quantifier phrases $(\forall x)$ and $(\exists x)$ are read as 'Every *object*' and 'Some *object*', or, equivalently, as 'Every*thing*' and 'Some*thing*', respectively. That is, the absolute quantifiers are understood as implicitly containing the most general or ultimate common name 'object' (which we take to be synonymous with 'thing').

The quantifier phrases $(\forall A)$ and $(\exists A)$ are taken as referring to every, or to some, name concept, respectively. Name constants are introduced in particular applications of the logic.

Because complex names contain formulas as relative clauses, *names* and *formulas* are inductively defined simultaneously as follows:

(1) every name variable or constant is a *name*;

(2) for all objectual variables x, y, $(x = y)$ is a *formula*; and

(3) if φ, ψ are formulas, B is a name (complex or simple), and x and C are an objectual and a name variable respectively, then $\neg\varphi$, $(\varphi \rightarrow \psi)$, $(\forall x)\varphi$, $(\forall x B)\varphi$, and $(\forall C)\varphi$ are *formulas*, and

(4) B/φ and $/\varphi$ are *names*, where $/\varphi$ is read as 'object that is φ'.

Among the rules, or meaning postulates, of our logic of names are four that were mentioned in our previous lecture. The first two connect relative quantifier phrases with absolute phrases, and the next two amount to export and import rules for quantifier phrases with complex names.

(MP1) $(\forall xA)\varphi \leftrightarrow (\forall x)[(\exists yA)(x = y) \rightarrow \varphi]$,

(MP2) $(\exists xA)\varphi \leftrightarrow (\exists x)[(\exists yA)(x = y) \wedge \varphi]$,

(MP3) $(\forall xB/\varphi)\psi \leftrightarrow (\forall xB)[\varphi \rightarrow \psi]$,

(MP4) $(\exists xB/\varphi)\psi \leftrightarrow (\exists xB)[\varphi \wedge \psi]$.

The axioms of the simple logic of names are those of the free logic of identity plus the axioms for name quantifiers. We will not list the axioms here, but they can be found in section 3 of:

http://www.stoqnet.org/lat_notes.html, Course 50547: Elements of Formal Ontology, Lecture 7.

The universal instantiation law in free logic for object variables is a theorem schema of this logic; that is, where x, y are distinct variables and y is free for x in φ, the following is provable:

$$(\exists x)(x = y) \rightarrow [(\forall x)\varphi \rightarrow \varphi(y/x)].$$

The following also are theorems of our conceptualist logic:
T1: $(\exists x A)\varphi \leftrightarrow (\exists x)[(\exists y A)(x = y) \wedge \varphi]$
T2: $(\exists x A/\psi)\varphi \leftrightarrow (\exists x A)[\psi \wedge \varphi]$
T3: $(\forall x/\psi)\varphi \leftrightarrow (\forall x)[\psi \rightarrow \varphi]$
T4: $(\forall x)\varphi \rightarrow (\forall x A)\varphi$
T5: $(\forall x A)\varphi \rightarrow [(\exists z A)(y = z) \rightarrow \varphi(y/x)]$,
where y is free for x in φ.
T6: $(\exists x A)(y = x) \rightarrow (\exists x)(y = x)$
T7: $(\forall x)\varphi \leftrightarrow (\forall A)(\forall x A)\varphi$,
where A is not free in φ.

The following "comprehension principle," (CP_N):

$$(\forall A)(\exists B)(\forall x)[(\exists y B)(x = y) \leftrightarrow (\exists y A)(x = y)]$$

is also provable and amounts to a form of existential generalization for name concepts A regardless of the complexity of A.

4. The Consistency and Decidability of the Simple Logic of Names

The simple logic of names that we have formulated in the previous section is both consistent and decidable. This holds because the logic is actually equiconsistent with monadic predicate logic, which is known to be consistent and decidable. We will not go through the details of showing this here. Those details can be found on the website already mentioned.

5. A Conceptualist Interpretation of Leśniewski's System

We now turn to a translation of Leśniewski's logic of names, as briefly described in section 2, into our conceptualist logic of names. We assume that the name variables a, b, c, d (with or with out numerical subscripts) of Leśniewski's logic are correlated one-to-one with the name variables A, B, C, D (with or without numerical subscripts) of our conceptualist logic.

That is, we assume that A is correlated with a, B is correlated with b, C is correlated with c, etc.

Because the only atomic formulas of the system are of the form '$a \varepsilon b$', the following inductive definition of a translation function trs translates each formula of Leśniewski's logic into a formula of our conceptualist logic (with a replaced by A, b by B, etc.):

1. $trs(a \varepsilon b) = (\forall xA)(\forall yA)(x = y) \wedge$
   $(\exists xA)(\exists yB)(x = y)$,
2. $trs(\neg\varphi) = \neg trs(\varphi)$,
3. $trs(\varphi \to \psi) = [trs(\varphi) \to trs(\psi)]$,
4. $trs((\forall a)\varphi) = (\forall A)trs(\varphi)$.

Note that the two conjuncts in clause (1) together are equivalent to saying that exactly one thing is A (and hence one thing is a) and that thing is a B, i.e., the thing that is a is a b, which is how Leśniewski understood '$a\ \varepsilon\ b$' as singular inclusion.

Note also that where φ is a logical axiom of the first-order logic of Leśniewski's system, then $trs(\varphi)$ is a theorem of our conceptualist logic. Modus ponens and (UG) also preserve validity under trs. We need now only prove that trs translates the axioms of Leśniewski's logic into a theorem of our present system.

For example, both of the axioms, (Compl) and (Conj), of Leśniewski's logic — one stipulating that every name has a complementary name, and the other that there is a name corresponding to the conjunction of singular inclusion in any two names — can be derived from the comprehension principle (CP_N) of our conceptualist logic.

The derivation of the translation of Leśniewski's principal axiom, which is the only one remaining, is relatively trivial, but long on details, and we will not go into those detail here. In any case, we have the following metatheorem.

Metatheorem 6: If φ is a theorem of Leśniewski's (first-order) logic of names, then $trs(\varphi)$ is a theorem of our conceptualist simple logic of names.

6. Explaining the Oddities of Leśniewski's Logic of Names

Finally, let us turn to an explanation of the oddities of Leśniewski's logic of names, i.e., an explanation in terms of our translation of Leśniewski's logic into our simple logic of names.

First, in regard to the seemingly implausible thesis,

$$(\exists a)(a \neq a),$$

of Leśniewski's logic, note that by Leśniewski's definition of identity (and hence of nonidentity) $(a \neq a)$ is really short for $\neg(a\ \varepsilon\ a \wedge a\ \varepsilon\ a)$, which is equivalent to $\neg(a\ \varepsilon\ a)$.

On our conceptualist interpretation, this formula translates into

$$\neg[(\forall xA)(\forall yA)(x = y) \wedge (\exists xA)(\exists yA)(x = y)],$$

which in effect says that *it is not the case that exactly one thing is an* A, a thesis that is provable in our conceptualist logic when A is taken as a necessarily vacuous common name, such as 'object that is not self-identical', which is symbolized as '$/(x \neq x)$', or more fully as '$Object/(x \neq x)$'.

Thus, the translation of Leśniewski's thesis,

$$(\exists a)(a \neq a),$$

is equivalent in our conceptualist logic of names to the provable thesis that there is a name concept A for which it is not the case that exactly one thing is A.

Note also that because $(a = b)$ in Leśniewski's system means $(a\ \varepsilon\ b \wedge b\ \varepsilon\ a)$, then the translation of $(a = b)$ into our conceptualist logic becomes

$$(\forall xA)(\forall yA)(x = y) \wedge (\exists xA)(\exists yB)(x = y) \wedge$$
$$(\forall xB)(\forall yB)(x = y) \wedge (\exists xB)(\exists yA)(x = y).$$

What this formula says is that exactly one thing is A and that thing is B, and that exactly one thing is B and that thing is A, a statement that is true when A and B are proper names, or unshared common names, of the same thing, and false otherwise, which is exactly how Leśniewski understood the formula $(a = b)$.

The form of existential generalization that we found odd in Leśniewski's logic, namely,

$$\varphi(c/a) \rightarrow (\exists a)\varphi(a),$$

is translated into our conceptualist logic as:

$$\varphi(C/A) \rightarrow (\exists A)\varphi(A),$$

which, if C is free for A in φ, is provable in our simple logic of names, and yet, of course, from this it does not follow, as it does in Leśniewski's logic, that something is identical with nothing.

In regard to the following thesis of Leśniewski's logic,

$$a \ \varepsilon \ b \rightarrow a \ \varepsilon \ a,$$

note that its translation into our conceptualist logic is,

$$(\forall xA)(\forall yA)(x = y) \wedge (\exists xA)(\exists yB)(x = y) \rightarrow \\ (\forall xA)(\forall yA)(x = y) \wedge (\exists xA)(\exists yA)(x = y),$$

which says that if exactly one thing is an A and that thing is a B, then exactly one thing is an A and that thing is an A, which is unproblematic.

Similarly, the translation, which we will avoid writing out in full here, of Leśniewski's seemingly odd transitivity thesis,

$$a \ \varepsilon \ b \wedge b \ \varepsilon \ c \rightarrow a \ \varepsilon \ c,$$

says that if exactly one thing is an A and that thing is a B and that if exactly one thing is a B (which therefore is the one thing that is an A), then exactly one thing is A and that thing is C. This thesis is easily seen to be valid in our conceptualist logic.

7. On the Use of Proper and Common Names

The apparent oddities of Leśniewski's logic of names are the result of treating both proper and common names as if they were "singular terms," i.e., expressions that can be substituends of object variables and occur as arguments (subjects) of predicates.

That, in any case, is how they are understood in Leśniewski's elementary ontology as an applied first-order logic (without identity). Of course, that is also how proper names, but not common names, are usually analyzed in modern logic.

But then, before the development of free logic where proper names that denote nothing are allowed, it was sometimes also the practice to transform proper names into monadic predicates. The proper name 'Socrates', for example, became the monadic predicate 'Socratizes', which was true of exactly one thing, and the name 'Pegasus' became 'Pegasizes', which was true of nothing. In this way, the statement that Pegasus does not exist could be analyzed as saying that nothing Pegasizes.

Common names, on the other hand, have usually been analyzed as, or really transformed into, monadic predicates in modern logic, both before and after the development of free logic.

Of course, in our general framework of conceptual realism there are complex monadic predicates that are constructed on the basis of both proper and common names. Thus, where A is a name, proper or common, then

$$[\lambda x(\exists yA)(x = y)]$$

is a monadic predicate, read as 'x is an A' when A is a common name, and as 'x is A' when A is a proper name.

Leśniewski's logic of names is viewed as odd in modern logic, as we have said, because it takes common names to be more like proper names than like monadic predicates, and in particular it represents them the way that "singular terms" are represented in modern logic. On this view, if common names were to be put in the same syntactic category as proper names, then it should be by taking both as monadic predicates.

Now in our conceptualist logic, proper names and common names are in the same syntactic category, but it is not the category of monadic predicates, nor is it the category of "singular terms".

Proper and common names belong to a more general category of names, and as such they are taken as parts of quantifier phrases, i.e., phrases that stand for referential concepts. This is not at all like taking them as "singular terms," the way they are in Leśniewski's logic, though, as we will explain shortly, they can be transformed into object terms, i.e., terms that can be substituends of object variables and occupy the argument positions of predicates.

The important point is that unlike the view of names in Leśniewski's logic, the occurrence of names as parts of quantifier phrases is an essential component of how the nexus of predication is understood in conceptual realism. Having a single category of names containing both proper and common names is a basic part of our theory of reference.

This does not mean that we cannot distinguish a proper name from a common name. In fact, when a proper name is introduced into a formal language we add a meaning postulate to the effect that the name can be used to refer to at most one thing, which is how proper names are understood in natural language. Common names, on the other hand, are not introduced into an applied formal language with such a meaning postulate.

Now there are other uses of proper and common names as well. Both, for example, can be used in *simple acts of naming*, as when a parent teaches a child what a dog or a cat is by pointing to the animal and saying 'dog', or 'cat'.

A simple act of naming is not an assertion and does not involve the exercise of either a predicable or a referential concept.

Also, names, both proper and common, can be used in greetings, or in exclamations as when someone shouts 'Wolf!'or 'Fire!', which again are not assertions and do not involve the exercise of a referential act.

A common name such as 'poison' is also used as a label, which again is not a referential act. Nor are referential acts involved in the use of name labels that people wear at conferences.

These kinds of uses of names, especially proper names or sortal common names, i.e., names that have identity criteria associated with their use, are conceptually prior to the referential use of names in sentences. The analysis of these kinds of uses as well as the use of names in referential acts belongs to the discipline of pragmatics, and not semantics, which deals exclusively with denotation and truth conditions.

8. Classes as Many as the Extensions of Names

There are also "denotative" uses of names as well, as when we speak of **mankind**, by which we mean the totality, or entire group, of humans taken collectively — but not in the sense of a set or class as an abstract object.

Thus, we say that Socrates is a member of mankind, as well as that Socrates is a man. Also, instead of 'mankind', we can use the plural of 'man' and say that Socrates is one among men.

This in fact is a transformation of the name 'man' into a "term," an expression that can occur as an argument of predicates. But it is not a "singular term" in the sense that it denotes a single entity, e.g., a set or a class as an abstract object.

Instead of using the phrase 'singular term', which suggests that we are dealing with a "single" entity, a better, or less misleading, phrase is 'object term', which we have used instead. An "object" such as mankind is not a "single" entity, but **a plural object**, i.e., **a plurality** taken collectively.

Now the transformation of 'man' into 'mankind', or 'human' into 'humankind', and 'dog' into 'dogkind', etc. is different from the nominalizing transformation of a predicate adjective, such as 'human', into an abstract noun — i.e., an abstract "singular term" — such as 'humanity'.

The transformation of a predicate, e.g., 'is human', into an abstract noun, 'humanity', results in a genuine "singular term", i.e., a term that purports to denote a single object, albeit an abstract intensional one.

But the transformation of 'man' into 'mankind', or 'dog' into 'dogkind', does not result in an expression that purports to denote a single object; nor does it purport to denote an abstract intensional object.

What such a noun as 'mankind', or 'dogkind', purports to denote is **a plural object**, namely, men, or dogs, taken collectively as a group — but not as a set or a class as a single object.

The expressions 'mankind' and 'dogkind', are indeed "nominal" expressions, i.e., nouns, and therefore, logically, they should be represented as "object terms," but not as "singular" terms in the sense of nominal expressions that denote single objects. What they denote are pluralities, i.e., plural objects.

A plural object, such as a group of things, is what Bertrand Russell once called **a class as many**, as opposed to **a class as one**.

Russell allowed that a class as many could consist of just a single object, as when a common name has just one object in its extension, in which case the class as many is the same as that one object. On the other hand, there is no class as many that is empty.

There is more than one notion of a class, in other words, and in fact there is even more than one notion of a class in the sense of the iterative concept of a set, i.e., the concept of a set based on Cantor's power-set theorem.

The iterative concept of a set can be developed, for example, either with an axiom of foundation or an axiom of *anti*-foundation. But in neither case can there be a universal set, and yet there are set theories, such as Quine's NF and the related set theory NFU, in which there is a universal set.

In fact, the notion of a universal class is part of the traditional, logical notion of a class as the extension of a predicable concept, and, as we have noted in a previous lecture, this was how classes were understood by Frege in his *Grundgetsetze.*

Now our point here is that classes in all of these senses are single objects, not plural objects, i.e., they are each a class as one, a single abstract object.

It is not the notion of a class as one, i.e., as a single abstract object, that we are concerned with here, but the notion of a class as many, i.e., of a class as a plurality, or plural object. It is this notion that is implicitly understood as the extension of a common count noun, or what we have been calling a common name.

In the development of our analysis of this notion, we will also take a class as many consisting of just one object as the extension of a nonvacuous proper name.

Membership in a class as many can be defined once names are allowed to be "nominalized" and occur as object terms. The definition is as follows:

$$x \in y =_{\mathbf{df}} (\exists A)[y = A \wedge (\exists zA)(x = z)],$$

where A is a name variable or constant. Note that in this definition, A occurs as a "nominalized", object term in the conjunct '$y = A$' as well as part of the quantifier phrase in the second conjunct '$(\exists zA)(x = z)$'.

An obvious theorem of the logic of classes as many, which we will develop in the next lecture, is the following,

$$x \in A \leftrightarrow (\exists z A)(x = z),$$

where '$x \in A$' can be read as 'x is an A', or 'x is one among the A', or 'x is a member of the class as many of A', or simply as 'x is a member of A'.

As understood by Russell, there are three important features of the notion of a class as many as the extension of a common name. These are:

(1) A vacuous common name, i.e., a common name that names nothing, has no extension, which is not the same as having an empty class as its extension. Thus, according to Russell, "there is no such thing as the null class, though there are null class-concepts," i.e., common-name concepts that have no extension.

(2) The extension of a common name that names just one thing is just that one thing. Unlike the singleton sets of set theory, which are not identical with their single member, the class that is the extension of a common name that names just one thing is none other than that one thing.

(3) Unlike sets, classes as the extensions of names are literally made up of their members so that when they have more than one member they are in some sense pluralities (Vielheiten), or "plural objects," and not things that can themselves be members of classes.

Thus, according to Russell, "though terms [i.e., objects] may be said to belong to ... [a] class, the class [as a plurality] must not be treated as itself a single logical subject."

It is a class as many that is the extension, or denotatum, of a common name, and, on our analysis, also of a proper name. On our analysis, **the logic of classes as many is a direct and natural extension of the simple logic of names** described earlier in this lecture. The idea is that names, both proper and common, can be transformed into, or "nominalized" as, object terms that can be substituends of object variables and occur as arguments of predicates.

When so transformed, what a name denotes is its extension, which in the case of a common name with more than one object in its extension is **a plural object**, which we will also call **a group**.

The extension of a proper name, on the other hand, is the object, if any, that the name denotes as a "singular" term.

The resulting logic of classes as many is not entirely unlike the analysis given in Leśniewski's logic of names, where names are represented as object terms. In fact we can even formulate counterparts in this logic to certain of the oddities of Leśniewski's logic.

But there is also a difference in that the counterparts of Leśniewski's problematic oddities are refutable, and the counterparts that are not refutable do not appear as odd but as natural consequences of an ontology with both single and plural objects.